# CCR 2018 – Thirteenth International Conference on Computability, Complexity and Randomness.

Universidad Andres Bello and CMM – U. de Chile

December 12, 2018

# Contents

# Invited Talks

## Computational complexity of attractors/repellers in one dimensional real and complex dynamics.

Monday
9:30am–10:30am

Michael Yampolsky

Department of Mathematics, University of Toronto.

The study of the quadratic family $z^2 + c$ has been a cornerstone of one-dimensional dynamics. Julia sets of these maps are among the most drawn images in Mathematics due both to their beauty, and their intrinsic complexity. We have previously shown with M. Braverman, that even for computable values of c, some of these sets are not computable, and have characterized the non-computable examples. Many delicate open questions have remained regarding computational complexity of computable quadratic Julia sets. I will briefly review the recent progress and will present a powerful new technique for constructing lower complexity bounds. It originated in our recent work with C. Rojas, which tackled a natural related question of computational complexity of attractors in the family of real quadratics, viewed as mappings of the real line.

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## Complexity, dynamics of subshifts, and algebraic properties.

Monday
10:30am–11:30am

Bryna Kra

Department of Mathematics, Northwestern University.

One way to classify dynamical systems is by their entropy, which roughly speaking gives a measure of the disorder in the system, and within zero entropy, we have a finer classification via the complexity of the system. We discuss the relations between these measurements in subshifts, dynamical characteristics of the system (such as periodicity, minimality, and transitivity), combinatorial problems, and algebraic properties of the system.

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Monday
2:30pm–3:30pm

## Computing the homology of semi-algebraic sets.


Felipe Cucker

Department of Mathematics, City University of Hong Kong.



We describe a new algorithm to compute all the homology groups of a semi-algebraic set. This algorithm is numerically stable and has weak exponential complexity. This means that outside a set of data of exponentially small measure its cost is singly exponential. This is in sharp contrast with state-of-the-art algorithms for this problem, which have a doubly exponential cost for all data outside a set of measure zero.


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Tuesday
9:30am–10:30am

## Expansions in bases 2 and 3: old conjectures and new results.


Pablo Shmerkin

Departamento de Matemáticas, U. de Torcualto Di Tella, Argentina



In the 1960s, H. Furstenberg proposed a series of conjectures that aimed to capture the heuristic principle that "expansions in bases 2 and 3 have no common structure". I will discuss a sample of these conjectures and the recent resolution of some of them. Although these problems lie at the intersection of ergodic theory and fractal geometry, no background on either area will be assumed.


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Tuesday
2:30pm–3:30pm

## Complexity of nilsystems and systems lacking of nilfactors.


Alejandro Maass

Departamento de Matemáticas, CMM-DIM, U. de Chile



Nilsystems are a natural generalization of rotations and arise in various contexts, including in the study of multiple ergodic averages in ergodic theory, in the structural analysis of topological dynamical systems, and in asymptotics for patterns in certain subsets of the integers. We show, however, that many natural classes in both measure preserving systems and topological dynamical systems contain no higher order nilsystems as factors, meaning that the only nilsystems they contain as factors are rotations. Our main result is that in the topological setting, nilsystems have a particular type of complexity of polynomial growth, where the polynomial (with explicit degree) is an asymptotic both from below and above. We also deduce several ergodic and topological applications of these results.


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## Algorithmic Dimensions of Projected Points.

Wednesday
9:30am–10:30am

Neil Lutz

Computer and Information Science, Penn-State Univiersity

To what extent are fractal dimensions preserved by projection mappings? In this talk, we will consider an effective and pointwise version of this question in the Euclidean plane. I will describe recent progress on this question and show how it has yielded new results about the classical Hausdorff dimension of projected sets, including new extensions to Marstrand's projection theorem.

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## Semi-computable geometry.

Thursday
9:30am–10:30am

Mathieu Hoyrup

INRIA — LORIA, Nancy, France.

Computability and semicomputability of compact subsets of the Euclidean spaces are important notions, that have been investigated for many classes of sets including fractals (Julia sets, Mandelbrot set) and objects with geometrical or topological constraints (embedding of a sphere). In this work we investigate one of the simplest classes, namely the filled triangles in the plane. We study the properties of the parameters of semicomputable triangles, such as the coordinates of their vertices, which is a surprisingly rich problem. We introduce and develop a notion of semicomputability of points of the plane which is a generalization in dimension 2 of the left-c.e. and right-c.e. real numbers, and relate this notion to Solovay reducibility. We show that semicomputable triangles admit no finite parametrization, for some notion of parametrization. We will also present more recent progress on this topic by investigating semicomputability in ordered vector spaces. This is joint work with Diego Nava Sauceo and Donald Stull.

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## Online structures

Thursday
2:30pm–3:30pm

Keng Meng Ng

Nanyang Technological University, Singapore.

We give a survey of the foundations of online structure theory. This work has its roots in the early work on efficiently computable structures, particularly polynomial-time computable structures. We compare and contrast with the situation for computable structures and show that online structures often behave in surprising ways.

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Friday
9:30am–10:30am

# Degrees of non-computability of points in general spaces

Takayuki Kihara

Department of Mathematical Informatics, Nagoya University, Japan.

In this talk, we survey our recent work on generalizations of degrees of non-computability. In particular, we examine an analogue of tt-reducibility for points in computable metric spaces. We characterize the notion of the metric tt-degree in the context of a first-level Borel isomorphism. Then, we study this concept from the perspective of effective topological and fractal dimension theory.

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# Invited Tutorials

## A Computer theoretical outlook on foundations of quantum information.

Tuesday and Wednesday 10:30am–11:30am

Santiago Figueira

Universidad de Buenos Aires, Argentina.

We introduce non-locality, a phenomenon by which measuring a property of a quantum system can instantaneously determine the results of another property measured on a distant system. We study Bell tests and its connection to non-locality and non-signaling distributions. We also present some notions of computability and randomness. We study what happens when we amalgamate notions of learnability, randomness and computability on the one hand and Bell scenarios for testing non-locality on the other. We present a new loophole for Bell-like experiments. We prove that choosing the inputs for a Bell tests using private pseudorandom number generators allows an adversary to prepare local boxes that pretend to be non-local. We also explain why deterministic hidden-variable models of non-local correlations need to be uncomputable if we want to prevent those correlations from being signaling.

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## Topological dynamics associated to Turing Machines.

Thursday and Frida
10:30am–11:30am

Anahí Gajardo

Universidad de Concepción, Chile.

Since 90's Turing machines are studied as topological dynamical systems, providing very interesting and complex examples. Now we know several of their properties and how are they related, we expect to give the basics together with an overview of the state of the art.

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# Contributed Talks

## Computability at zero temperature

Monday
11:30am–12pm

Christian Wolf
City University of New York

Let $f : X \to X$ be a one-dimensional subshift of finite type. In this talk, we discuss the computability of certain thermodynamic invariants that are associated with a continuous potential $\phi : X \to \mathbb{R}$. In particular, we consider the residual entropy (i.e., the joint ground state entropy) and the zero-temperature measure (i.e., the limit of equilibrium states when the temperature goes to 0) of the system. We show that the residual entropy $h_{\infty,\phi}$ is an upper semi-computable function of the potential $\phi$, but it is not computable. Moreover, $\phi \mapsto h_{\infty,\phi}$ is computable at $\phi_0$ if and only of $h_{\infty,\phi_0} = 0$. Next, we consider locally constant potentials for which the zero-temperature measure $\mu_{\infty,\phi}$ is known to exist. We characterize the computability of the zero-temperature measure and its entropy for potentials that are constant on cylinders of a given length $k$. In particular, we show the existence of an open and dense set of locally constant potentials $\phi$ for which the zero-temperature measure $\mu_{\infty,\phi}$ can be computationally identified as an elementary periodic point measure. Finally, we show that our methods do not generalize to treat the case when $k$ is not given. The results presented in this talk are joint work with Michael Burr (Clemson University), see arXiv:1809.00147 [math.DS].

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## The domino problem for word-hyperbolic groups

Monday
3:30pm–4pm

Sebastián Barbieri
University of Bristish Columbia

The domino problem for a finitely generated group asks whether there exists an algorithm which receives on input a finite alphabet and a coding of a finite number of forbidden patterns and decides whether there is a tiling of the group by the alphabet which avoids the forbidden patterns. We show that such an algorithm exists for a word-hyperbolic group if and only if it is virtually free.

This is joint work with Nathalie Aubrun and Étienne Moutot.

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Monday
4:30pm–5pm

## Computability of invariant measures of minimal Cantor systems

Alexander Frank, Cristobal Rojas, Mathieu Hoyrup and Daniel Coronel.
UNAB and INRIA LORIA

It is known that the set of invariant measures of a compact dynamical system is a generalized simplex, a so-called Choquet simplex. A Choquet simplex is such that any of its elements can be written as a generalized combination of its extreme points in a unique way.

In 1991, T. Downarowicz proved that every Choquet simplex can be realized within the class of Toeplitz sub-shifts in two symbols, and therefore within the class of minimal Cantor systems.

Here we exhibit a finite-dimensional computable version: Given any finite-dimensional upper-computable simplex $K$, there is a computable minimal Cantor system such that its set of invariant measures is computable affine homeomorphic to $K$.

As a corollary, we show that there exist computable minimal Cantor systems with finitely many ergodic measures such that none of them is computable.

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Monday
5pm–5:30pm

## The word complexity and how to compute it

Sebastien Ferenczi, Christian Mauduit and Carlos Gustavo Moreira.
CNRS, Aix-Marseille Université and IMPA

The complexity function of an infinite word counts the number of its factors. For any positive function $f$, its *exponential rate of growth* $E_0(f)$ is $\liminf_{n\to\infty} \frac{1}{n}\log f(n)$. We define a new quantity, the *word entropy* $E_W(f)$, as the maximal exponential growth rate of a complexity function smaller than $f$. This is in general smaller than $E_0(f)$, and more difficult to compute; we give an algorithm to estimate it. We use $E_W(f)$ to compute the Hausdorff dimension of the set of real numbers whose expansions in a given base have complexity bounded by $f$.

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Tuesday
3:30pm–4pm

## Selection, Divergence, and Dichotomy

Xiang Huang, Jack H. Lutz, Elvira Mayordomo and Donald Stull.
Iowa State Unviersity

The Schnorr-Stimm dichotomy theorem concerns finite-state gamblers that bet on infinite sequences of symbols taken from a finite alphabet. The theorem asserts that, for each such sequence S, the following two things are true. 1. If S is normal in the sense of Borel (meaning that any two strings of equal length appear with equal asymptotic frequency in S), then every finite-state gambler loses money at an exponential rate betting on S. 2. If S is not normal, then there is a finite-state gambler that wins money at an exponential rate betting on S. In this paper we use the Kullback-Leibler divergence (also known as the relative entropy) to generalize the dichotomy theorem to arbitrary probability measures on the alphabet and to quantify the exponential rates of winning and losing on the two sides of the dichotomy.

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## A conditional inequality in Kolmogorov complexity: some remarks and applications in communication complexity

Tuesday
4:30pm–5pm

Marius Zimand
Towson University

It is well-known that upon conditioning mutual information can increase or decrease. That is, there are $x, y, t_1, t_2$ such that $I(x : y|t_1) > I(x : y)$ and $I(x : y|t_2) < I(x : y)$. This is valid both in classical information theory where mutual information is based on Shannon entropy, and in algorithmic information where mutual information is based on Kolmogorov complexity. In this note we use the framework of Kolmogorov complexity.

Romashchenko and Zimand have shown that if t is a computable function of x and y and if furthermore this function has the "rectangle property" stating that $t(x_1, y_1) = t(x_2, y_2) = t$ implies $t(x_1, y_2) = t(x_2, y_1) = t$, then conditioning with $t(x, y)$ is guaranteed to decrease mutual information: $I(x : y|t(x, y)) \leq I(x : y) + O(\log n)$.

A related result using Shannon entropy instead of Kolmogorov complexity, has been obtained by Kaced et al.

The center piece of this note is an extension of the Romashchenko-Zimand result, which is applicable to nondeterministic communication complexity. We show several examples of such applications.

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## Schnorr randomness and Levy's Upward Theorem

Tuesday
5pm–5:30pm

Francesca Zaffora Blando
Stanford University

The main result of our paper is a characterization of Schnorr randomness in terms of an effectivization of Lévy's Upward Theorem:

**Theorem 1** *Let $\omega \in 2^{\mathbb{N}}$. The following are equivalent:*

1. *$\omega$ is Schnorr random;*
2. *for all fast computable $L^1$-Cauchy sequences $\xi_n$ with $\xi_n \to \xi$ in $L^1$, $\lim_{n\to\infty} \xi_n(\omega)$ exists and $\lim_{n\to\infty} \xi_n(\omega) = \lim_{\ell\to\infty} \mathbb{E}[\xi \mid \mathfrak{F}_\ell](\omega)$.*

This is the natural Cantor space analogue of a result in Euclidean space of Pathak, Rojas, and Simpson.

In the setting of formal epistemology, it is natural to think about elements of Cantor space as representing possible outcomes or worlds. The integrable random variable, by contrast, is a quantity which we are interested in determining, and restriction to computable random variables is reflective of the restrictions on our epistemic resources. Under this interpretation, Theorem 1 says that the world is Schnorr random if and only if, for all such computable quantities, our method of approximating the value of the quantity at the world eventually aligns with the true value of that quantity.

In the paper, we also investigate other martingale convergence theorems. For instance, we look at an effective version of Lévy's Downward Theorem. We consider the decreasing sequence of sub-$\sigma$-algebras whose intersection corresponds to the Borel subsets of Cantor space which are closed under the $E_0$-equivalence relation from descriptive set theory, and we restrict attention to $L^1$-computable random variables. We then show that this effective version of Lévy's Downward Theorem corresponds to Schnorr randomness.

This is joint work with Simon Huttegger (UCI) and Sean Walsh (UCLA).

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Wednesday
11:30am–12pm

## A weak randomness notion for probability measures on Cantor space

Andre Nies and Frank Stephan.

The University of Auckland and NUS

We consider an algorithmically defined randomness notion for Borel probability measures measures on Cantor space. The notion is weaker than the usual one using the uniform measure on the space of probability measures. This research interacts with a recent attempt to define ML-randomness for quantum states corresponding to infinitely many qubits.

A. Nies is supported in part by the Marsden Fund of the Royal Society of New Zealand, UoA 13-184. F. Stephan is supported in part by the Singapore Ministry of Education Academic Research Fund Tier 2 grant MOE2016-T2-1-019 / R146-000-234-112.

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Thursday
11:30am–12pm

## A Framework for Random Structures

Russell Miller

Queens College and the Graduate Center – CUNY

We present a framework under which it is reasonable to characterize certain countable structures as "random." In doing so, we extend work by Khoussainov. The framework applies to several particular classes of structures, including subrings of the rational numbers, algebraic field extensions of the rationals, finite-branching infinite trees, and finite-valence connected pointed graphs. We will discuss possible extensions to other classes, and also will explain how Martin-Lof randomness can be used to infer certain properties of structures in some of these classes.

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## Computable categoricity and randomness

Thursday
3:30pm–4pm

Johanna Franklin

Hofstra University

R. Miller has recently determined a method for placing a topology on the spaces of the isomorphism types of computable algebraic structures such as algebraically closed fields and finitely branching trees. This allows us to introduce measures on these spaces in a natural way, which gives us a new way to discuss the frequency of certain properties of computable structures, including computable categoricity and relative computable categoricity.

In fact, we can do more: we can define randomness for these isomorphism types and, for any measure-one property, we can ask whether there is a type of randomness that will guarantee it. In the case of algebraically closed fields of characteristic 0, R. Miller and I have shown that Schnorr randomness is enough to guarantee uniform computable categoricity. We will present this result as well as some results on finitely branching trees.

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## The Wadge ordering on the Scott domain is not a well quasi-ordering

Thursday
4:30pm–5pm
P1

Louis Vuilleumier and Jacques Duparc

Université de Lausanne

If $\mathcal{X}$ is a topological space and $\mathcal{A}, \mathcal{B} \subseteq \mathcal{X}$, $\mathcal{A}$ is continuously reducible to $\mathcal{B}$, written $\mathcal{A} \leq_w \mathcal{B}$, if there exists a continuous function $f : \mathcal{X} \to \mathcal{X}$ such that $f^{-1}(\mathcal{B}) = \mathcal{A}$. The Scott domain is the set $\mathcal{P}(\mathbb{N})$ endowed with the topology generated by the basis $\big\{\{x \subseteq \mathbb{N} \mid F \subseteq x\} \mid F \subseteq \mathbb{N} \text{ finite}\big\}$. We show that the quasi-order induced by continuous reductions on the Scott domain is ill-founded and has infinite antichains, and that these properties already occur within the $\boldsymbol{\Delta}^0_2$ subsets, i.e. at very low levels of topological complexity. The main result is solely due to the second author who used tools developed by the first author.

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Thursday
5pm–5:30pm
P1

# Classifying $\Delta^0_2$ equivalence relations via computable reducibility

Luca San Mauro

Vienna University of Technology

Computable reducibility is a long-standing notion that allows to compare the complexity of equivalence relations over the natural numbers. In this paper, we rely on the Ershov hierarchy to study the degree structure induced by computable reducibility on $\Delta^0_2$ equivalence relations. Our goal is to understand to which extent results holding for c.e. equivalence relations can be generalized: we offer many disanalogies between these contexts and explain why dealing with $\Delta^0_2$ equivalence relations requires novel techniques. In particular, we focus on the problem of the existence of infima and suprema, and we prove that if $\mathcal{X} \in \{\Pi^{-1}_a : |a|_{\mathcal{O}} \geq 2\} \cup \{\Sigma^{-1}_a : |a|_{\mathcal{O}} \geq 3\}$, then the structure of $\mathcal{X}$-equivalence relations is elementarily equivalent to neither c.e. equivalence relations nor co-c.e. equivalence relations.

This is joint work with N. Bazhenov, M. Mustafa, A. Sorbi, and M. Yamaleev.

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Thursday
5:30pm–6pm
P1

# A domain theoretical study of total computable functions

Claudio Callejas and Benjamín Callejas Bedregal

Federal University Rural do Semi-Árido (UFERSA) and Federal University of Rio Grande do Norte (UFRN)

The partial and total non-necessary computable functions in $\omega$, have been studied from a topological (see [3, 4, 5]) and domain theoretical (see [3, 5]) approach, however, the partial and total computable functions, only from a topological point of view (see for instance [2]). In this talk, based on [1], the total computable and non-computable functions will be studied from a domain theoretical perspective, which gives us the associated Scott topologies that are compared with the existing topologies. The regular computable functions play a fundamental role in the total computable functions definition's (in recursion theory), but its study has been neglected in the literature. As a consequence of the results that will be presented in this talk we obtain a new one for the regular computable functions.

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## A proof of the existence of an aperiodic 2-symbol reversible and complete Turing machine

Thursday
4:30pm–5pm
P2

Rodrigo Torres-Avilés
Universidad del Bío-Bío

Since Kurka conjecture in 1997 about the non-existence of aperiodic Turing machine in TMT model, counter examples has been presented in distinct and more restricted class of Turing machines, the last one a 3-symbol Complete and Reversible Turing machine in 2017. In this work we formally present a proof of aperiodicity of a 2-symbol 8-states complete and reversible Turing machine, created by Julien Cassaigne. Also, this machine is studied in several other related properties, as transitivity and minimality.

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## Genericity and randomness with ITTM's

Thursday
5pm–5:30pm
P2

Benoit Monin and Paul-Elliot Angles d'Auriac
UPEC - LACL

The study of Infinite-Time Turing machines, ITTMs for short, goes back to a paper by Hamkins and Lewis. Informally these machines work like regular Turing machines, with in addition that the time of computation can be any ordinal. Special rules are then defined to specify what happens at a limit step of computation.

This simple computational model yields several new non-trivial classes of objects, the first one being the class of objects which are computable using some ITTM. These classes have been later well understood and characterized by Welch. ITTMs are not the first attempt of extending computability notions. This was done previously for instance with alpha-recursion theory, an extension of recursion theory to $Sigma_1$-definability of subsets of ordinals, within initial segments of the Godel constructible hierarchy. Even though alpha-recursion theory is defined in a rather abstract way, the specialists have a good intuition of what "compute" means in this setting, and this intuition relies on the rough idea of "some" informal machine carrying computation times through the ordinal. ITTMs appeared all the more interesting, as they consist of a precise machine model that corresponds to part of alpha-recursion theory.

Recently Carl and Schlicht used the ITTM model to extend algorithmic randomness and effective genericity notions in this setting. Genericity and randomness are two different approaches to study typical objects, that is, objects having "all the typical properties" for some notion of typicality. For randomness, a property is typical if the class of reals sharing it is of measure 1, whereas for genericity, a property is typical if the class of reals sharing it is co-meager. In both cases, for any countable collection of typical properties, it is still a typical property to share all of them: the intersection of countably many measure-one sets is still a measure-one set, and the intersection of countably many co-meager sets is still a co-meager set. Depending on the countably many properties we consider, the reals that share all of them may be of great interest, in forcing constructions or to study various notions of degrees, from Turing to alpha-degrees.

We will present a general framework to study randomness and genericity within Godel's constructible hierarchy. Using this framework, we will answer several of Carl and Schlicht's open questions and we will ask new ones. We believe in particular that one of this question, yet unanswered, is difficult and interesting : It is a question about the separation of two randomness notions defined by Carl and Schlicht. It seems so clear at first that the two notions should be different, that the question has not been asked so far. The reason is certainly that the analogues of this two notions in Higher randomness actually differ for simple reasons. We will emphasize that things are not so simple in the settings of ITTMs, and we will show how the two notions are much closer than one might think.

This question motivates the study of Cohen genericity notions with respect to ITTM : In order to argue that it is not absurd to think that these two randomness notions may actually coincide, we will show that it is the case for their categorical analogues. The versions of these analogues with Higher genericity are also known to differ for simple reasons, like it is the case with randomness. We believe that the proof of equality of these two genericity notions is also interesting in itself, as it uses the new phenomenons that occur within some levels of the constructible hierarchy.

A paper on this has recently been submitted to JSL, and can be found here: `https://www.lacl.fr/~benoit.monin/ressources/papers/genericity_and_randomness_in_ITTMs.pdf`

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Thursday
5:30pm–6pm
P2

## The Effective Dimensions of Points on Lines

Donald Stull
Université de Lorraine, CNRS, INRIA

This paper is concerned with the algorithmic dimension of points on a given line in the Euclidean plane. The most well-studied algorithmic dimensions for a point $x \in R^n$ are the *effective Hausdorff dimension*, $\dim(x)$, and its dual, the *effective packing dimension*, $Dim(x)$. In this paper we investigate the (effective) dimension spectra of lines in the Euclidean plane. The dimension spectrum of a line $L_{a,b}$, $\mathrm{sp}(L)$, with slope $a$ and intercept $b$ is the set of all effective dimensions of the points $(x, ax+b)$ on $L$.

It has been recently shown that, for every $a$ and $b$ with effective dimension less than 1, the dimension spectrum of $L_{a,b}$ contains an interval. Our first main theorem shows that this holds for every line. Moreover, when the effective dimension of $a$ and $b$ is at least 1, $\mathrm{sp}(L)$ contains a *unit* interval. Our second main theorem gives lower bounds on the dimension spectra of lines. In particular, we show that for every $\alpha \in [0,1]$, with the exception of a set of Hausdorff dimension at most $\alpha$, the effective dimension of $(x, ax+b)$ is at least $\alpha + \frac{\dim(a,b)}{2}$. As a consequence of this theorem, using a recent characterization of Hausdorff dimension using effective dimension, we give a new proof of a result by Molter and Rela on the Hausdorff dimension of Furstenberg sets.

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# Post's Programme Revisited

Friday
11:30am–12pm
P2

Rod Downey
Victoria University

We characterize when $A \leq_{ibT} M$ for a maximal $M$. This introduces a new hierarchy to approximate c.e. sets. joint work with Klaus Ambos-Spies and Martin Monath.